Fermions tunnelling from the charged dilatonic black holes

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Abstract

Kerner and Mann's recent work shows that, for an uncharged and non-rotating black hole, its Hawking temperature can be correctly derived by fermions tunnelling from its horizons. In this paper, our main work is to improve the analysis to deal with charged fermion tunnelling from the general dilatonic black holes, specifically including the charged, spherically symmetric dilatonic black hole, the rotating Einstein-Maxwell-Dilaton-Axion (EMDA) black hole and the rotating Kaluza-Klein (KK) black hole. As a result, the correct Hawking temperatures are well recovered by charged fermions tunnelling from these black holes.

Keywords: Hawking radiation, Tunnelling, fermions, charged black hole.

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1 Introduction

Since Hawking proved that a black hole can radiate thermally [1], many papers have appeared to deeply discuss the quantum radiation of black holes via different methods [2]. Recently, Wilczek and his collaborators have proposed two universal methods to correctly recover Hawking radiation of black holes. In Ref.[3], when the classically irrelevant ingoing modes at the event horizon of the black hole is neglected, the effective chiral theory contains an anomaly with respect to general coordinate symmetry, which is often named as gravitational anomaly. To cancel gravitational anomaly and restore general coordinate covariance at the quantum level, an energy momentum tensor flux must be introduced at the horizon. The result shows that the compensating energy momentum tensor flux has an equivalent form as that of $(1+1)$-dimensional blackbody radiation at the Hawking temperature. Later, much work further promotes this method to the cases of different-type black holes[4, 5, 6].

On the other hand, Hawking radiation could be viewed as a semi-classical quantum tunnelling process. According to the WKB approximation, the tunnelling rate takes the form as $\Gamma \propto \exp(-2\mathrm{Im}I)$, where I is the classical action of the trajectory. Thus the calculation of the imaginary part of the action becomes most important for this tunnelling method. In general, two universal methods are applied in references to derive the action. One method is called as the Null Geodesic method(a detail description is available in Ref.[7]), and regarding the imaginary part of the action as only contribution of the momentum p_r of the emitted null s-wave. Another method, proposed by Srinivasan and Padmanabhan[8] and recently developed by Angheben et.al[9], is successfully present to derive the imaginary part of the action by solving the Hamilton-Jacobi equation, which is, later, called as the Hamilton-Jacobi method.

Till now, a lot of work has already been successfully carried out for further development of the tunnelling approach, but all of them are only focused on Hawking radiation of scalar particles from various black hole spacetimes [10, 11]. In fact, a black hole can radiate any types of particles at the Hawking temperature, and the true emission spectrum should contain contributions of particles with charge and all possible spins. Recently, Kerner and Mann have succeeded to apply uncharged fermion tunnelling from a non-rotating black hole to correctly recover its Hawking temperature [12]. Subsequently, people extend the analysis to the cases of Kerr black hole, Kerr-Newman black hole and dynamical horizon and all the results are satisfying[13]. In this paper, we further improve it to deal with charged

fermion tunnelling from the general dilatonic black holes, specifically including the charged, spherically symmetric dilatonic black hole, the rotating Einstein-Maxwell-Dilaton-Axion (EDMA) black hole and the rotating Kaluza-Klein(KK) black hole.

The rest of the paper are organized as follows. In Sec.2, we study charged fermion tunnelling from the charged, spherically symmetric dilatonic black hole, and the expected Hawking temperature is well recovered. In Sec.3 and Sec.4, for a broad extension, we again check charged fermions tunnelling from the rotating Einstein-Maxwell-Dilaton-Axion (EMDA)and Kaluza-Klein (KK) black holes. Sec.5 contains some discussions and conclusions.

2 Fermions tunnelling from the charged, spherically symmetric dilatonic black hole

In this section, we focus our attention on Hawking radiation of fermions via tunnelling from the charged spherically symmetric dilatonic black hole. Dilaton field is a kind of scalar field which occurs in the low energy limit of the string theory where the Einstein action is supplemented by fields such as axion, gauge fields and dilaton coupling in a nontrivial way to the other fields. Solutions for charged dilaton black holes in which the dilaton is coupled to the Maxwell field have been obtained and have many differences from that of ordinary black holes obtained in the Einstein gravitational theory. Since the presences of dilaton they have important consequences on the causal structure and the thermodynamic properties of the black hole, and much interest is attracted to study the dilaton black holes. The spherically symmetric solution [14] is obtained from the four-dimensional low energy Lagrangian

$$S=\int dx^4\sqrt{-g}\left[-R+2\left(\nabla\Phi\right)^2+e^{-2a\Phi}F^2\right], \tag{1}$$

where a is a parameter, and denotes the strength of the coupling of the dilation field Φ to Maxwell field F. When $a=0$, it reduces to the usual Einstein-Maxwell scalar theory. When $a=1$, it is part of the low energy action of string theory. The metric of the charged, spherically symmetric dilatonic black hole (also called as G. H dilatonic black hole) reads as

$$\begin{aligned}
&ds^2=-e^{2U}\left(r\right)dt^2+e^{-2U}\left(r\right)dr^2+R^2\left(r\right)\left(d\theta^2+\sin^2\theta d\varphi^2\right),\\
&e^{2\Phi}=\left(1-\frac{r_-}{r}\right)^{\frac{2a}{1+a^2}},\quad F=\frac{Q}{r^2}dt\wedge dr,
\end{aligned} \tag{2}$$

where $e^{2U}(r) = \left(1-\frac{r_h}{r}\right)\left(1-\frac{r_-}{r}\right)^{\frac{1-a^2}{1+a^2}}$, $R(r) = r\left(1-\frac{r_-}{r}\right)^{\frac{a^2}{1+a^2}}$, Φ and F are the dilaton and Maxwell fields, the mass and electric charge of the black hole are expressed as $M = \frac{r_h}{2} + \frac{r_-}{2}\cdot\frac{1-a^2}{1+a^2}$ and $Q^2 = \frac{r_h r_-}{1+a^2}$, respectively. r_h/r_- are the outer/inner horizon of the black hole, a is a couple constant confined in $0 \le a < 1$. When $a = 0$, this metric reduces to the Reissner-Nordström solution. The electric potential of the G. H. dilatonic black hole is $A_\mu = A_t dt = \frac{Q}{r}dt$. For all a, it is singular at the location of the outer horizon $r = r_h$.

The motion equation of charged fermion in the electromagnetic field can be written as

$$i\gamma^\mu\left(\partial_\mu + \Omega_\mu + \frac{i}{\hbar}eA_\mu\right)\psi + \frac{m}{\hbar}\psi = 0, \tag{3}$$

where $\Omega_\mu = \frac{i}{2}\Gamma_\mu^{\alpha\beta}\sum_{\alpha\beta}$, $\sum_{\alpha\beta} = \frac{i}{4}\left[\gamma^\alpha, \gamma^\beta\right]$ and γ^μ matrices satisfy $\{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu} \times I$, m and e are the mass and the electric charge of the emitted particles, and A_μ is the electric potential of the black hole. To deal with fermions tunnelling radiation, it is important to choose an appropriate γ^μ matrices. There are many ways to choose them[12, 13], in our case, we choose

$$\begin{aligned}
\gamma^t &= \frac{1}{\sqrt{e^{2U}(r)}}\begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^\theta = \frac{1}{R(r)}\begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}, \\
\gamma^r &= \sqrt{e^{2U}(r)}\begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^\phi = \frac{1}{R(r)\sin\theta}\begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix}.
\end{aligned} \tag{4}$$

Here, σ^i is the Pauli Sigma matrices. For fermion with spin 1/2, the wave function has two spin states (namely, spin up($\uparrow$) and down ($\downarrow$) states), so we can take the following ansatz as

$$\begin{aligned}
\Psi_\uparrow &= \begin{pmatrix} A(t,r,\theta,\varphi) \\ 0 \\ B(t,r,\theta,\varphi) \\ 0 \end{pmatrix} \exp\left(\frac{i}{\hbar}I_\uparrow(t,r,\theta,\varphi)\right), \\
\Psi_\downarrow &= \begin{pmatrix} 0 \\ C(t,r,\theta,\varphi) \\ 0 \\ D(t,r,\theta,\varphi) \end{pmatrix} \exp\left(\frac{i}{\hbar}I_\downarrow(t,r,\theta,\varphi)\right),
\end{aligned} \tag{5}$$

where $\Psi_\uparrow$ denotes the wave function of spin up particle, and $\Psi_\downarrow$ is for spin down case. Inserting Eq.(5) for spin up particle into the Dirac equation and dividing the exponential term and multiplying by $\hbar$, we have

$$-\left(\frac{iA}{\sqrt{e^{2U}(r)}}\left(\partial_t I_\uparrow + eA_t\right) + B\sqrt{e^{2U}(r)}\partial_r I_\uparrow\right) + mA = 0, \tag{6}$$

$$\left(\frac{iB}{\sqrt{e^{2U}\left(r\right)}}\left(\partial_t I_\uparrow + eA_t\right) - A\sqrt{e^{2U}\left(r\right)}\partial_r I_\uparrow\right) + mB = 0, \tag{7}$$

$$\frac{B}{R\left(r\right)}\partial_\theta I_\uparrow + \frac{iB}{R\left(r\right)\sin\theta}\partial_\varphi I_\uparrow = 0, \tag{8}$$

$$\frac{A}{R\left(r\right)}\partial_\theta I_\uparrow + \frac{iA}{R\left(r\right)\sin\theta}\partial_\varphi I_\uparrow = 0. \tag{9}$$

It is difficult to directly solve the action from above equations. Considering the symmetries of the space-time, we can carry out separation of variables for the action as

$$I_\uparrow = -\omega t + W\left(r\right) + \Theta\left(\theta,\varphi\right), \tag{10}$$

where ω is the energy of the emitted particle. Then substituting Eq.(10) into Eqs.(6)and (7), the radial function $W\left(r\right)$ satisfies the following equations

$$\left(\frac{iA}{\sqrt{e^{2U}\left(r\right)}}\left(\omega - eA_t\right) - B\sqrt{e^{2U}\left(r\right)}\partial_r W\left(r\right)\right) + mA = 0, \tag{11}$$

$$-\left(\frac{iB}{\sqrt{e^{2U}\left(r\right)}}\left(\omega - eA_t\right) + A\sqrt{e^{2U}\left(r\right)}\partial_r W\left(r\right)\right) + mB = 0. \tag{12}$$

Here, we neglect the equations about $\Theta\left(\theta,\varphi\right)$ function. Although $\Theta\left(\theta,\varphi\right)$ could provide a contribution to the imaginary part of the action, its total contributions to the tunnelling rate are cancelled out [12]. At the event horizon, the radial function $W\left(r\right)$ can be written as

$$\begin{aligned} W_\pm\left(r\right) &= \pm\int \frac{\sqrt{\left(\omega - eA_t\right)^2 + m^2 e^{2U}\left(r\right)}}{e^{2U}\left(r\right)}dr \\ &= \pm i\pi r_h\left(\omega - eA_0\right)\left(1 - \frac{r_-}{r_h}\right)^{\frac{a^2-1}{a^2+1}}, \end{aligned} \tag{13}$$

where $+/-$ correspond to the outgoing/ingoing solutions, and $A_h = Q/r_h$ is the electric potential at the event horizon. So the tunnelling probability of charged fermion is

$$\begin{aligned} \Gamma &= \frac{P_{(emission)}}{P_{(absorption)}} = \frac{\exp(-2\mathrm{Im}I_{\uparrow+})}{\exp(-2\mathrm{Im}I_{\uparrow-})} = \frac{\exp(-2\mathrm{Im}W_+)}{\exp(-2\mathrm{Im}W_-)} \\ &= \exp\left(-4\pi r_h\left(\omega - eA_0\right)\left(1 - \frac{r_-}{r_h}\right)^{\frac{a^2-1}{a^2+1}}\right), \end{aligned} \tag{14}$$

which means Hawking temperature of the dilatonic black hole is

$$T = \frac{1}{4\pi r_h}\left(1 - \frac{r_-}{r_h}\right)^{\frac{1-a^2}{1+a^2}}. \tag{15}$$

Now Hawking temperature has been correctly derived via fermion with spin up tunnelling from the dilatonic black hole. For spin down case, the similar process is adopted and the same result can be recovered. When $a = 0$, the metric (2) is the solution of the Reissner-Nordström black hole, and Hawking temperature is recovered from Eq.(15) as

$$T = \frac{1}{2\pi}\frac{\sqrt{M^2 - Q^2}}{\left(M + \sqrt{M^2 - Q^2}\right)^2}. \tag{16}$$

In the extreme limit $r_h = r_-$, it is found that the surface gravity is zero and Hawking temperature of the black holes (the extreme Reissner-Nordström black hole and the extreme $U(1)$ dilatonic black hole) vanishes for all $0 \le a < 1$. But for $a = 1$ the surface gravity is a constant; and for $a > 1$, it divergences because the black hole approaches its extremal limit[15], at the time the nearly extremal black hole behaves more like elementary particles.

3 Fermions tunnelling from the rotating EMDA black hole

In this section, we study charged fermions tunnelling from the rotating Einstein-Maxwell-Dilaton-Axion(EMDA) black hole. In 1995, Alberto García et.al gave a class of stationary axisymmetric solutions of the Einstein-Maxwell-Dilaton-Axion field equations. From the action

$$\begin{aligned} S &= \int dx^4\sqrt{-g}[R - 2g^{\mu\nu}\partial_\mu\phi\partial_v\phi \\ &- \frac{1}{2}e^{4\phi}g^{\mu\nu}\partial_\mu\kappa\partial_v\kappa - e^{-2\phi}F_{\mu\nu}F^{\mu\nu} - \kappa F_{\mu\nu}\breve{F}^{\mu\nu}], \end{aligned} \tag{17}$$

the solution of the EMDA black hole[16] can be obtained as

$$\begin{aligned} ds^2 &= -\frac{\Delta - a^2\sin^2\theta}{\sum}dt^2 - \frac{2a\sin^2\theta\,(r^2 + 2br + a^2 - \Delta)}{\sum}dtd\varphi \\ &+ \frac{\sum}{\Delta}dr^2 + \sum d\theta^2 + \frac{(r^2 + 2br + a^2)^2 - \Delta a^2\sin^2\theta}{\sum}\sin^2\theta d\varphi^2, \end{aligned} \tag{18}$$

where

$$\begin{aligned} &A_\mu = A_t dt + A_\varphi d\varphi = \frac{Qr}{\sum}dt - \frac{Qra\sin^2\theta}{\sum}d\varphi, \\ &\sum = r^2 + 2br + a^2\cos^2\theta, \\ &\Delta = r^2 - 2mr + a^2 = (r - r_h)(r - r_-), \\ &r_h = m + \sqrt{m^2 - a^2}, r_- = m - \sqrt{m^2 - a^2}. \end{aligned} \tag{19}$$

The dilaton ϕ and axion scalar κ fields are, respectively, given as $\exp(2\phi_0) = \omega$ and $\kappa = \kappa_0$, where ω and κ_0 are constants, and r_h/r_- are the outer/inner horizons of the black hole. The parameters m, a and b are the mass, angular momentum per unit mass and dilatonic constant of the black hole, which are related to the ADM mass M, charge Q and angular momentum J of the black hole with

$$M = m + b, \quad Q^2 = 2b(m + b), \quad J = (m + b) a. \tag{20}$$

When $a = 0$, the EMDA metric reduces to the Garfinkle-Horowitz-Strominger dilatonic solution. When $b = m \sinh^2\left(\frac{\alpha}{2}\right)$ and $\omega = 1$, one can derive the parameters of characterizing the Kerr-Sen black hole. For simplicity of our computation, we introduce the dragging coordinate transformation as $\phi = \varphi - \Omega t$, where

$$\Omega = \frac{a\left(r^2 + 2br + a^2 - \Delta\right)}{\left(r^2 + 2br + a^2\right)^2 - \Delta a^2 \sin^2\theta}, \tag{21}$$

to the metric (18), then the new metric takes the forms as

$$ds^2 = -f(r)\, dt^2 + \frac{1}{g(r)} dr^2 + \sum d\theta^2 + g_{33} d\phi^2, \tag{22}$$

where

$$f(r) = \frac{\Delta \sum}{\left(r^2 + 2br + a^2\right)^2 - \Delta a^2 \sin^2\theta}, \quad g(r) = \frac{\Delta}{\sum},$$
$$g_{33} = \frac{\left(r^2 + 2br + a^2\right)^2 - \Delta a^2 \sin^2\theta}{\sum} \sin^2\theta. \tag{23}$$

The corresponding potential is

$$\mathcal{A}_\mu = \mathcal{A}_t dt + \mathcal{A}_\phi d\phi = \frac{\left(r^2 + 2br + a^2\right) Qr}{\left(r^2 + 2br + a^2\right)^2 - \Delta a^2 \sin^2\theta} dt - \frac{Qra \sin^2\theta}{\sum} d\phi. \tag{24}$$

In order to solve Dirac equation, we must first introduce the metrics γ^μ. As mentioned in Sec.2, there are many different ways to choose them. Considering the similarity between the metrics (2) and (22), we choose γ^μ matrices as

$$\gamma^t = \frac{1}{\sqrt{f(r)}} \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \quad \gamma^\phi = \frac{1}{\sqrt{g_{33}}} \begin{pmatrix} 0 & \sigma^2 \\ \sigma^2 & 0 \end{pmatrix},$$
$$\gamma^r = \sqrt{g(r)} \begin{pmatrix} 0 & \sigma^3 \\ \sigma^3 & 0 \end{pmatrix}, \quad \gamma^\theta = \frac{1}{\sqrt{\sum}} \begin{pmatrix} 0 & \sigma^1 \\ \sigma^1 & 0 \end{pmatrix}. \tag{25}$$

We still choose the same form as given in Eq.(5) for the wave functions of charged fermion tunnelling from the EMDA black hole, and only explore spin up case. Substituting the γ^μ matrices (25) and the wave function into Dirac equation (3), we have

$$-\left(\frac{iA}{\sqrt{f(r)}}\left(\partial_t I_\uparrow + e\mathcal{A}_t\right) + B\sqrt{g(r)}\partial_r I_\uparrow\right) + mA = 0, \tag{26}$$

$$\left(\frac{iB}{\sqrt{f(r)}}\left(\partial_t I_\uparrow + e\mathcal{A}_t\right) - A\sqrt{g(r)}\partial_r I_\uparrow\right) + mB = 0, \tag{27}$$

$$\frac{B}{\sqrt{\Sigma}}\partial_\theta I_\uparrow + \frac{iB}{\sqrt{g_{33}}}\left(\partial_\phi I_\uparrow + e\mathcal{A}_\phi\right) = 0, \tag{28}$$

$$\frac{A}{\sqrt{\Sigma}}\partial_\theta I_\uparrow + \frac{iA}{\sqrt{g_{33}}}\left(\partial_\phi I_\uparrow + e\mathcal{A}_\phi\right) = 0. \tag{29}$$

There are four equations, but our interest is the first two ones because the tunnelling rate is directly related to the imaginary part of the radial function, and the angular contribution can be cancelled out when dividing the outgoing probability by the ingoing probability. In view of the properties of the rotating EMDA space-time, one can carry out separation of variables for the action as

$$I_\uparrow = -\left(\omega - j\Omega\right)t + W(r) + j\phi + \Theta(\theta), \tag{30}$$

where ω is the energy of the emitted particle for the observer at the infinity, and j is the angular quantum number about φ. Inserting the action (30) into Eqs. (26) and (27) yields

$$\left(\frac{iA}{\sqrt{f(r)}}\left(\omega - j\Omega - e\mathcal{A}_t\right) - B\sqrt{g(r)}\partial_r W(r)\right) + mA = 0, \tag{31}$$

$$-\left(\frac{iB}{\sqrt{f(r)}}\left(\omega - j\Omega - e\mathcal{A}_t\right) + A\sqrt{g(r)}\partial_r W(r)\right) + mB = 0. \tag{32}$$

Here exists two cases. When $m = 0$, the above equations describe the radial wave function for the massless particle. When $m \neq 0$, charged massive fermion is in consideration, and solving the above equations yields

$$\begin{aligned} W_\pm(r) &= \pm\int\sqrt{\frac{\left(\omega - j\Omega - e\mathcal{A}_t\right)^2 + m^2 f(r)}{f(r)\,g(r)}}dr \\ &= \pm i\pi\frac{\omega - j\Omega_h - e\mathcal{A}_t(r_h)}{\sqrt{f'(r_h)\,g'(r_h)}}, \end{aligned} \tag{33}$$

where $+(-)$ correspond to the outgoing (ingoing) solutions, and $\Omega_h = \Omega(r_h) = a/\left(r_h^2 + 2br_h + a^2\right)$ is the angular velocity at the event horizon of the EMDA black hole. Thus the tunnelling probability of charged fermion can be written as

$$\begin{aligned} \Gamma &= \frac{P_{(emission)}}{P_{(absorption)}} = \frac{\exp(-2\mathrm{Im}I_{\uparrow +})}{\exp(-2\mathrm{Im}I_{\uparrow -})} = \frac{\exp(-2\mathrm{Im}W_{+})}{\exp(-2\mathrm{Im}W_{-})} \\ &= \exp\left(-4\pi \frac{\omega - j\Omega_h - e\mathcal{A}_t(r_h)}{\sqrt{f'(r_h)\, g'(r_h)}}\right). \end{aligned} \tag{34}$$

According to the relationship between the tunnelling rate and Hawking temperature, Hawking temperature of the EMDA black hole can be obtained as

$$T = \frac{\sqrt{f'(r_h)\, g'(r_h)}}{4\pi} = \frac{1}{2\pi}\frac{r_h - m}{r_h^2 + 2br_h + a^2}. \tag{35}$$

When $b = 0$, Hawking temperature T can be written as

$$T = \frac{1}{2\pi}\frac{\sqrt{m^2 - a^2}}{\left(m + \sqrt{m^2 - a^2}\right)^2 + a^2}, \tag{36}$$

which is Hawking temperature of the Kerr black hole. For $b = 0$ and $a = 0$ (Schwarzschild black hole case), Hawking temperature equals $1/8\pi m$. So fermions tunnelling from these black hole can correctly recover Hawking temperatures.

4 Fermions tunnelling from the rotating Kaluza-Klein black hole

The solution of the rotating Kaluza-Klein black hole [17] denoted a five dimensional spacetime with a translational symmetry in a spacelike direction in four dimensional metrics $g_{\mu\nu}$ is obtained from the action (1) with $a = \sqrt{3}$, which reads

$$\begin{aligned} ds^2 &= -\frac{\Delta - a^2\sin^2\theta}{\Pi\sum}dt^2 + \frac{\Pi\sum}{\Delta}dr^2 + \Pi\sum d\theta^2 \\ &+ \left[\Pi\left(r^2 + a^2\right) + \frac{\mathrm{Z}}{\Pi}a^2\sin^2\theta\right]\sin^2\theta d\varphi^2 - \frac{2a\mathrm{Z}\sin^2\theta}{\Pi\sqrt{1 - v^2}}dtd\varphi, \end{aligned} \tag{37}$$

with the dilaton field $\Phi = -(\sqrt{3}\ln\Pi)/2$, and the electromagnetic potential

$$A_\mu = \frac{v}{2\left(1 - v^2\right)}\frac{\mathrm{Z}}{\Pi^2}dt - \frac{va\sin^2\theta}{2\sqrt{1 - v^2}}\frac{\mathrm{Z}}{\Pi^2}d\varphi, \tag{38}$$

where

$$
\begin{aligned}
&Z=\frac{2mr}{\sum}, \quad \Pi=\sqrt{1+\frac{v^2\mathrm{Z}}{1-v^2}},\\
&\sum=r^2+a^2\cos^2\theta, \quad \Delta=r^2-2mr+a^2,\\
&r_h=m+\sqrt{m^2-a^2}, \quad r_-=m-\sqrt{m^2-a^2}.
\end{aligned}
\tag{39}
$$

and a and v are the rotation parameter and boost velocity respectively. The outer(inner) horizons are described by the parameters $r_h(r_-)$, and satisfy $\Delta=0$. The solution reduces to the Kerr solution for $v=0$. The physics mass M, charge Q and angular momentum J of the black hole are related to the parameters m, a and v as

$$
M=\frac{m}{2}\cdot\frac{2-v^2}{1-v^2}, \quad Q=\frac{mv}{1-v^2}, \quad J=\frac{ma}{\sqrt{1-v^2}}. \tag{40}
$$

As mentioned in Sec.3, to easily investigate charged fermions tunnelling from the black hole, we first introduce the dragging coordinate transformation as $\phi=\varphi-\Omega t$, where

$$
\Omega=\frac{a\mathrm{Z}}{\left[\Pi^2\left(r^2+a^2\right)+\mathrm{Z}a^2\sin^2\theta\right]\sqrt{1-v^2}}. \tag{41}
$$

Then the new metric takes the form as

$$
ds^2=-F\left(r\right)dt^2+\frac{1}{G\left(r\right)}dr^2+\Pi\sum d\theta^2+g_{33}d\phi^2, \tag{42}
$$

with the new electromagnetic potential

$$
\begin{aligned}
\mathcal{A}_\mu &= \mathcal{A}_t dt+\mathcal{A}_\phi d\phi\\
&= \frac{Qr}{\sum}\frac{r^2+a^2}{\Pi^2\left(r^2+a^2\right)^2+\mathrm{Z}a^2\sin^2\theta}dt-\frac{va\sin^2\theta}{2\sqrt{1-v^2}}\frac{\mathrm{Z}}{\Pi^2}d\phi,
\end{aligned}
\tag{43}
$$

and

$$
\begin{aligned}
&F\left(r\right)=-\frac{\Pi\sum\Delta\left(1-v^2\right)}{\left(r^2+a^2\right)^2-\Delta\left(a^2\sin^2\theta+v^2\sum\right)},\\
&g_{33}=\left(\Pi\left(r^2+a^2\right)+\frac{\mathrm{Z}}{\Pi}a^2\sin^2\theta\right)\sin^2\theta, \quad G\left(r\right)=\frac{\Delta}{\Pi\sum}.
\end{aligned}
\tag{44}
$$

As the metric (42) takes the similar form as that of (22), we can choose the similar γ^μ matrices as Eq.(25) for the black hole, specifically takeing

$$
\begin{aligned}
&\gamma^t=\frac{1}{\sqrt{F\left(r\right)}}\begin{pmatrix} i & 0\\ 0 & -i\end{pmatrix}, \quad \gamma^\theta=\frac{1}{\sqrt{\Pi\sum}}\begin{pmatrix} 0 & \sigma^1\\ \sigma^1 & 0\end{pmatrix},\\
&\gamma^r=\sqrt{G\left(r\right)}\begin{pmatrix} 0 & \sigma^3\\ \sigma^3 & 0\end{pmatrix}, \quad \gamma^\phi=\frac{1}{\sqrt{g_{33}}}\begin{pmatrix} 0 & \sigma^2\\ \sigma^2 & 0\end{pmatrix}.
\end{aligned}
\tag{45}
$$

Substituting the above γ^{μ} matrices and the wave function with spin up given in (5) into Dirac equation yields

$$-\left(\frac{iA}{\sqrt{F(r)}}\left(\partial_t I_{\uparrow}+e\mathcal{A}_t\right)+B\sqrt{G(r)}\partial_r I_{\uparrow}\right)+mA=0, \tag{46}$$

$$\left(\frac{iB}{\sqrt{F(r)}}\left(\partial_t I_{\uparrow}+e\mathcal{A}_t\right)-A\sqrt{G(r)}\partial_r I_{\uparrow}\right)+mB=0, \tag{47}$$

$$\frac{B}{\sqrt{\Pi\sum}}\partial_\theta I_{\uparrow}+\frac{iB}{\sqrt{g_{33}}}\left(\partial_\phi I_{\uparrow}+e\mathcal{A}_\phi\right)=0, \tag{48}$$

$$\frac{A}{\sqrt{\Pi\sum}}\partial_\theta I_{\uparrow}+\frac{iA}{\sqrt{g_{33}}}\left(\partial_\phi I_{\uparrow}+e\mathcal{A}_\phi\right)=0. \tag{49}$$

Although there are four equations, our attention is also focused on the first two equations. Considering the properties of the Kaluza-Klein space-time, we carry out separation of variables as Eq.(30). Inserting the action $I_{\uparrow}$ into Eqs.(46) and (47) yields

$$\left(\frac{iA}{\sqrt{F(r)}}\left(\omega-j\Omega-e\mathcal{A}_t\right)-B\sqrt{G(r)}\partial_r W(r)\right)+mA=0, \tag{50}$$

$$-\left(\frac{iB}{\sqrt{F(r)}}\left(\omega-j\Omega-e\mathcal{A}_t\right)+A\sqrt{G(r)}\partial_r W(r)\right)+mB=0, \tag{51}$$

where ω denotes the energy of the emitted particles measured by the observer at the infinity, and j is the angular quantum number about φ. In the case $m \neq 0$, solving the above equations, we have

$$\begin{aligned} W_{\pm}(r) &= \pm\int\sqrt{\frac{\left(\omega-j\Omega-e\mathcal{A}_t\right)^2+m^2F(r)}{F(r)\,G(r)}}dr \\ &= \pm i\pi\frac{\omega-j\Omega_h-e\mathcal{A}_t(r_h)}{\sqrt{F'(r_h)\,G'(r_h)}}, \end{aligned} \tag{52}$$

where $+(-)$ signs represent the outgoing(ingoing) solutions, and $\Omega_h=\Omega(r_h)$ is the angular velocity at the outer horizon of the KK black hole. So the tunnelling probability of charged fermion of the black hole can be written as

$$\begin{aligned} \Gamma &= \frac{P_{(emission)}}{P_{(absorption)}}=\frac{\exp(-2\mathrm{Im}I_{\uparrow+})}{\exp(-2\mathrm{Im}I_{\uparrow-})}=\frac{\exp(-2\mathrm{Im}W_{+})}{\exp(-2\mathrm{I}mW_{-})} \\ &= \exp\left(-4\pi\frac{\omega-j\Omega_h-e\mathcal{A}_t(r_h)}{\sqrt{F'(r_h)\,G'(r_h)}}\right). \end{aligned} \tag{53}$$

Thus Hawking temperature of the Kaluza-Klein black hole takes the form as

$$T = \frac{\sqrt{F'(r_h)\,G'(r_h)}}{4\pi} = \frac{1}{4\pi} \cdot \frac{\sqrt{(1-v^2)(m^2-a^2)}}{m\left(m+\sqrt{m^2-a^2}\right)}. \tag{54}$$

When $v = 0$ and $Q = 0$, Hawking temperature of Eq.(54) equals Eq.(36), which means, in that case, the KK black hole can reduce to the Kerr black hole. And for $v = 0$ and $a = 0$, it describes a Schwarzschild black hole, and its Hawking temperature equals $1/8\pi m$. Obviously, the results once again proves the validity of the charged fermions tunnelling method.

5 Conclusions and Discussions

In this paper, we attempted to apply Kerner and Man's fermions tunnelling method to charged fermions' cases. As an example, Hawking radiation of charged fermions for a general charged, spherically symmetric dilatonic black hole is first studied via the tunnelling method. For a wide extension, charged fermions tunnelling from the rotating dilatonic black holes, specifically including the rotating Einstein-Maxwell-Dilaton-Axion (EDMA) and Kaluza-Klein (KK) black holes, are also considered in the paper. As a result, the correct Hawking temperatures are well described by charged fermions tunnelling from these black holes.

For simplicity to choose the metrics γ^μ when dealing with Hawking radiation of charged fermionss tunnelling from the rotating dilatonic black holes in Sec.(3) and Sec.(4), we carried out the dragging coordinate transformation. In fact, such behavior would not stop us from getting the correct Hawking temperature of these black holes, as in Ref.[12] to discuss Hawking radiation of black holes in the Painlevé and Kruskal-Szekers coordinate systems. In addition, after charged fermionss have tunnelled out, we assumed that the energy, charge and angular momentum of the dilatonic black holes keep the same as before. If the emitted particle's self-gravitational interaction is incorporated into Hawking radiation of fermionss tunnelling, Hawking temperatures will be corrected slightly, but their leading terms take the same form as Eqs.(16),(35) and (54).

In summary, we have succeeded in dealing with Hawking radiation of the rotating black holes via charged fermions tunnelling. This method can also be directly extended to the case of the non-stationary charged rotating black holes.

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